11 Beams and two-stream instability

Particle beam means a system of superthermal particles with narrow angular and velocity distribution.

Particle beams, especially electron beams, are quite common in the universe. Namely, they can be very easily generated. For example, if the collisionless plasma is at some location very rapidly heated, then electrons escape from this location and fast electrons escape to slower ones. Thus, after some time interval at some distance from the plasma heating region, the plasma distribution will consist of the background plasma electrons and a bump (electron beam) at velocities greater than local thermal velocity.

11.1 Beams in the solar atmosphere

There are many observations indicating the presence of particle beams in the solar atmosphere. Radio spectra in metric and decimetric frequency ranges present the most convincing cases. For example, the type III radio bursts (Fig. 49) are generated by electron beams. Along the beam trajectory due to the bump-on-tail instability Langmuir waves are generated. These waves are then transformed into electromagnetic ones, which escape from the solar corona and are observed by radiospectrographs. The frequencies of these radio bursts correspond to the local plasma frequencies in the solar atmosphere or their harmonics. Thus, if the electron beam is propagating upwards in the solar atmosphere, into the plasma with decreasing plasma densities, then the frequency of the observed radio emission is decreasing and vice versa. Because the typical speed of these electron beams is about one third of the speed of light and the height density scale of the solar corona is about $H_{ne} \simeq 5 \times 10^7$ m, the frequency drift of the type III burst is about 350 MHz s^{-1} in the 700 − 200 MHz frequency range.

Besides these radio observations there are further effects which reveal a presence of particle beams in the solar atmosphere. For example, hard X-ray and gamma-ray emissions and evaporation processes are explained by a bombardment of the chromosphere by superthermal electron, proton, or even neutral beams. (A neutral beam consists of both protons and electrons with the same density and same velocity.) Also in interpretations of asymmetries and polarizations of the optical chromospheric lines particle beams are assumed (Xu et al. 2005).

Furthermore, in-situ instruments onboard of satellites detect frequently particle beams of the solar origin in the heliosphere (Schwenn & Marsch 1991).

11.2 Two-stream instability

All calculations here, except for the kinetic beam instability, are made in the cold plasma approximation, which means that temperatures of plasma and beams are assumed to be zero. For more details, see Mikhailovskii (1975).

11.2.1 Dispersion equation

Let at times $t < 0$ a plasma exists in a stationary state, i.e. the plasma density, plasma velocity, magnetic and electric fields are:

$$n = n_0, \mathbf{v} = \mathbf{v}_0, \mathbf{B} = \mathbf{B}_0, \mathbf{E} = \mathbf{E}_0. \tag{367}$$

Then at times $t > 0$ small perturbations appear:

$$n = n_0 + n', \mathbf{v} = \mathbf{v}_0 + \mathbf{v}', \mathbf{B} = \mathbf{B}_0 + \mathbf{B}', \mathbf{E} = \mathbf{E}_0 + \mathbf{E}'. \tag{368}$$

Let us assume that these perturbations have periodic form in time as:

$$X'(t, \mathbf{r}) = X'(\mathbf{r}) \exp(-i\omega t), \tag{369}$$

where $\mathbf{r}$ is the position and ω is the frequency.

Then the mass conservation, momentum and Maxwell equations can be linearized. Thus, a set of equations for variables of zero-, first- and higher-orders of magnitude can be derived. The set of equations with first-order variables follows:

$$\begin{gathered}
-i\omega n' + \nabla \cdot (n'\mathbf{v}_0 + n_0\mathbf{v}') = 0, \\
-i\omega\mathbf{v}' + (\mathbf{v}_0 \cdot \nabla)\mathbf{v}' + (\mathbf{v}' \cdot \nabla)\mathbf{v}_0 = \frac{e}{m}(\mathbf{E}' + (\mathbf{v}' \times \mathbf{B}_0) + (\mathbf{v}_0 \times \mathbf{B}')), \\
\nabla \times \mathbf{B}' = \mu_0 e(n'\mathbf{v}_0 + n_0\mathbf{v}') - \frac{i\omega}{c^2}\mathbf{E}', \\
\nabla \times \mathbf{E}' = i\omega\mathbf{B}', \\
\nabla \cdot \mathbf{E}' = \frac{en'}{\epsilon_0}, \\
\nabla \cdot \mathbf{B}' = 0.
\end{gathered} \tag{370}$$

For the Fourier transform of differential operators, see Appendix.

11.2.2 Oscillations in homogenous plasma

Let us assume a 1-D case with $\mathbf{B}_0 = 0, \mathbf{v}_0 = 0, \nabla n_0 = 0$ and the spatial perturbation in the form: $X'(\mathbf{r}) \sim \exp(ik\mathbf{r})$. Then from the above mentioned equations follow:

$$\begin{gathered}
-i\omega n' + n_0 ikv' = 0, \\
-i\omega v' = \frac{e}{m}E', \\
ikE' = \frac{en'}{\epsilon_0}.
\end{gathered}$$

Now, from these equations the dispersion equation for so called plasma oscillations can be written as:

$$\omega^2 = \frac{e^2 n_0}{\epsilon_0 m}. \tag{371}$$

11.2.3 Electromagnetic waves in homogenous plasma

Furthermore we can write

$$\nabla \times \mathbf{B}' = \mu_0 e n_0 v' + \frac{1}{c^2}\frac{\partial \mathbf{E}'}{\partial t}, \quad / \frac{\partial}{\partial t}$$

$$\nabla \times \frac{\partial \mathbf{B}'}{\partial t} = \mu_0 e n_0 \frac{\partial v'}{\partial t} + \frac{1}{c^2}\frac{\partial^2 \mathbf{E}'}{\partial t^2},$$

$$-\nabla \times \nabla \times \mathbf{E}' = \mu_0 e n_0 \frac{e}{m}\mathbf{E}' + \frac{1}{c^2}\frac{\partial^2 \mathbf{E}'}{\partial t^2},$$

$$-k^2 \mathbf{E}' = \frac{\omega_p^2}{c^2}\mathbf{E}' - \frac{\omega^2}{c^2}\mathbf{E}',$$

$$\omega^2 = \omega_p^2 + k^2 c^2, \tag{372}$$

which is the dispersion equation for the electromagnetic waves in the cold plasma without static magnetic field.

11.2.4 Dispersion equation for plasma with moving components

Let us consider a potential perturbation of the electric field

$$\mathbf{E} = -\nabla\psi, \;\; \mathbf{E} = -i\mathbf{k}\psi,$$

and let us look what a perturbation of the electric charge density ρ_e causes the electric field perturbation, i.e. let us look for the function χ in the relation $\rho_e = \chi\psi$.

In this case the linearized MHD equation has a following form:

$$-i\omega n' + \nabla \cdot (n'\mathbf{v}_0 + n_0\mathbf{v}') = 0,$$

$$-i\omega\mathbf{v}' + (\mathbf{v}_0\nabla)\mathbf{v}' + (\mathbf{v}'\nabla)\mathbf{v}_0 = \frac{e}{m}\mathbf{E}.$$

Using $\nabla \cdot \mathbf{v}_0 = 0$ and $\mathbf{E} = -\nabla\psi$ the equation can be rewritten into

$$-i\omega n' + i n'\mathbf{k}\mathbf{v}_0 + i n_0\mathbf{k}\mathbf{v}' = 0,$$

$$-i\omega\mathbf{v}' + \mathbf{v}_0 i\mathbf{k}\mathbf{v}' = -i\frac{e}{m}\mathbf{k}\psi.$$

Now, we can express the density and plasma velocity perturbations as

$$n' = \frac{n_0\mathbf{k}\mathbf{v}'}{\omega - \mathbf{k}\mathbf{v}_0}, \quad \mathbf{v}' = \frac{e\mathbf{k}\psi}{m(\omega - \mathbf{k}\mathbf{v}_0)}.$$

Combining these equations the perturbation of charge density is

$$\rho_e = n'e = \frac{e^2 n_0 k^2}{m(\omega - \mathbf{k}\mathbf{v}_0)^2}\psi = \chi\psi. \tag{373}$$

Thus, the function χ for any specific plasma component α can be expressed as

$$\chi^\alpha = \frac{e^2 n_0^\alpha k^2}{m^\alpha(\omega - \mathbf{k}\mathbf{v}_0^\alpha)^2}. \tag{374}$$

Let us put these results into the Poisson equation. Then

$$k^2\psi = \frac{1}{\epsilon_0}\sum_\alpha \rho_e^\alpha,$$

$$k^2\psi = \frac{1}{\epsilon_0}\sum_\alpha \chi^\alpha\psi,$$

$$(1 - \frac{1}{k^2\epsilon_0}\sum_\alpha \chi^\alpha)\psi = 0. \tag{375}$$

In the last relation the term in brackets is the dispersion equation which can be formally written as

$$\hat{\epsilon}_0 = 1 - \frac{1}{k^2\epsilon_0}\sum_\alpha \chi^\alpha, \tag{376}$$

where contributions of moving components of plasma into the dispersion equation are

$$\hat{\epsilon}_0^\alpha = -\frac{1}{k^2\epsilon_0}\chi^\alpha = -\frac{(\omega_p^\alpha)^2}{(\omega - \mathbf{k}\mathbf{v}_0^\alpha)^2}. \tag{377}$$

11.3 Beam instabilities

11.3.1 Instability of two counter-streaming beams

Let us consider two counter-streaming beams of the same density, i.e.

$$n_{01} = n_{02} = n_0,\ v_{01} = -v_{02} = v. \tag{378}$$

In this case the dispersion equation is

$$1 - \frac{\omega_p^2}{(\omega - k_\parallel v)^2} - \frac{\omega_p^2}{(\omega + k_\parallel v)^2} = 0. \tag{379}$$

This equation leads to a bi-quadratic equation with the following solutions:

$$\omega = \pm\sqrt{(k_\parallel v)^2 + \omega_p^2 \pm \omega_p(\omega_p^2 + 4k_\parallel^2 v^2)^{1/2}}. \tag{380}$$

If

$$(k_\parallel v)^2 + \omega_p^2 < \omega_p(\omega_p^2 + 4k_\parallel^2 v^2)^{1/2},$$

i.e. if

$$(k_\| v)^4 + 2(k_\| v)^2\omega_p^2 + \omega_p^4 < \omega_p^2(\omega_p^2 + 4k_\|^2 v^2),$$

i.e. if $k_\| < \sqrt{2}\omega_p/v$ then there is one solution with $Im\ \omega > 0$, which for the perturbation in the form $X(t) \sim \exp^{-i\omega t}$ means an instability. Furthermore, if $k_\| \ll \omega_p/v$ then the term under the root can be written as

$$(k_\| v)^2 + \omega_p^2 - \omega_p^2(1 + \frac{2k_\|^2 v^2}{\omega_p^2}) = (k_\| v)^2 - 2(k_\| v)^2,$$

which gives the growth rate of the instability as

$$Im\ \omega = \mid k_\| v \mid . \tag{381}$$

On the other hand, the maximum growth rate can be derived as follows:

$$\frac{d}{dk_\|}((k_\| v)^2 + \omega_p^2 - \omega_p(\omega_p^2 + 4k_\|^2 v^2)^{1/2}) = 0,$$

$$2v^2 k_\| - \frac{\omega_p}{2}\frac{8k_\| v^2}{(\omega_p^2 + 4k_\|^2 v^2)^{1/2}} = 0,$$

$$\omega_p^2 + 4k_\|^2 v^2 = 4\omega_p^2,$$

$$k_{\| max} = \frac{\sqrt{3}}{2}\frac{\omega_p}{v}. \tag{382}$$

Now, putting this $k_{\| max}$ into the relation for ω (Eq. 380), the maximum growth rate is

$$\gamma_{max} = \omega_p/2. \tag{383}$$

11.3.2 Beam-plasma instability

Let us assume a beam which density is much lower than that of background plasma ($n_1 \ll n_0$). Then the dispersion equation is

$$1 - \frac{\omega_{pe}^2}{\omega^2} - \frac{\alpha\omega_{pe}^2}{(\omega - k_\| v)^2} = 0, \tag{384}$$

where $\omega_{pe}^2 = 4\pi e^2 n_0/m_e$, $\alpha = n_1/n_0 \ll 1$, v is the beam velocity.

Solutions:

a) The non-resonant case, i.e. the case with $\omega_{pe} \neq k_\| v$.